系統性質(property)

- Stability: 穩定性 — 實作系統必要之性質
- Memory: 記憶性
- Causality: 因果性 — 即時系統必要之性質
- Invertibility: 反系統
- Time invariance: 非時變
- Linearity: 線性

易於分析系統之基本性質

Stability: 穩定性

ee:p55

- BIBO: bounded-input, bounded-out, 一個系統對所有之有限輸入其輸出也是有限稱BIBO system

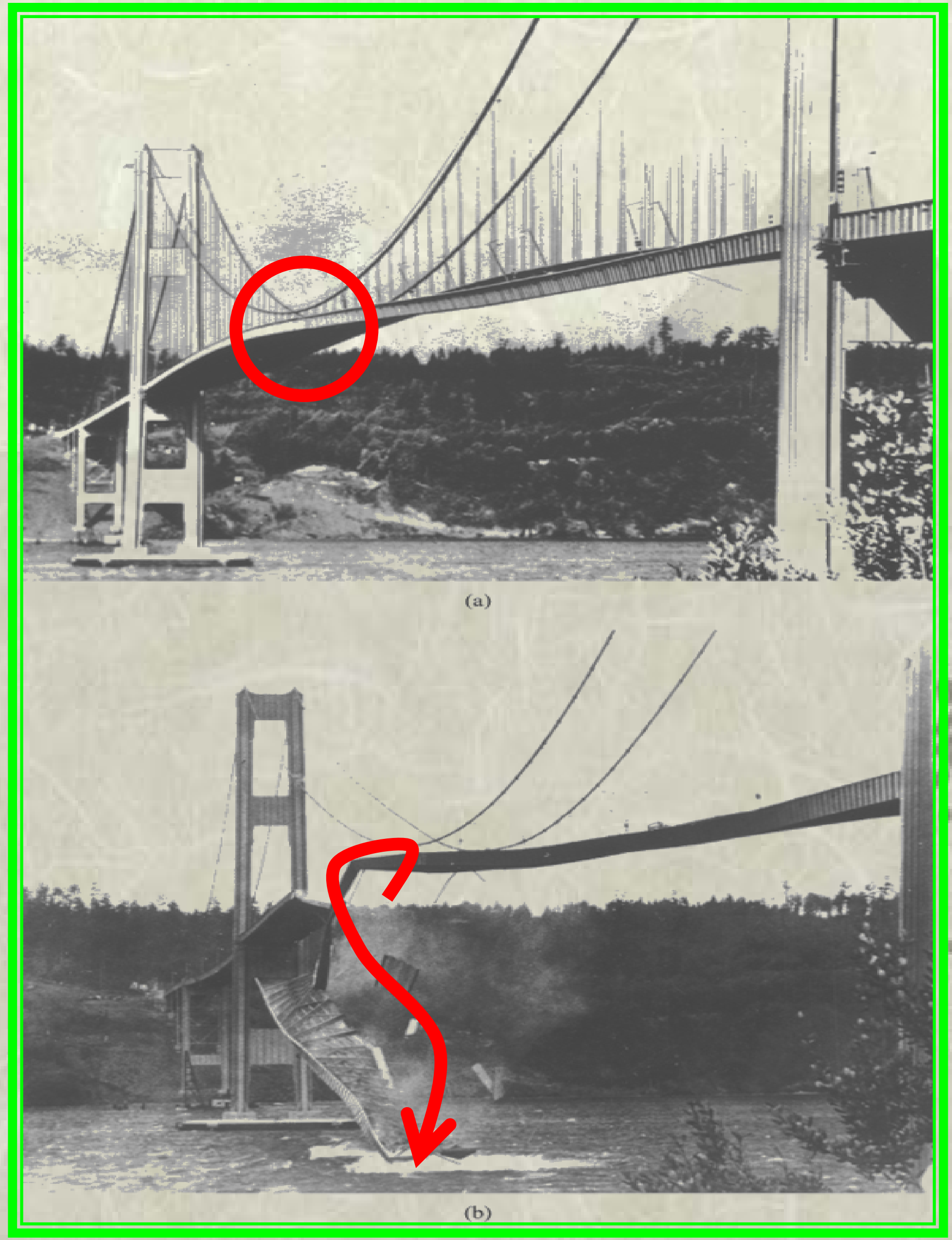
若 $y(t) = H\{x(t)\}$

對 $|x(t)| \le M_x < \infty$ for all t

滿足 $|y(t)| \le M_y < \infty$ for all t

稱H是BIBO穩定

Figure 1.52 (P. 56)



ee:p56

Ex 1.13 證明下列系統為BIBO穩定

ee:p55

$$y[n]=\frac{1}{3}\left(x[n]+x[n-1]+x[n-2]\right)$$

假設 $\left|x[n]\right|\le M_x<\infty$ for all n

$$\left|y[n]\right|=\left|\frac{1}{3}\left(x[n]+x[n-1]+x[n-2]\right)\right|$$

$$\le\frac{1}{3}\left(\left|x[n]\right|+\left|x[n-1]\right|+\left|x[n-2]\right|\right)$$

$$\le\frac{1}{3}\left(M_x+M_x+M_x\right)=M_x<\infty$$

$\therefore H$為BIBO穩定

ee:p57

Ex 1.14

$$y[n] = r^n x[n], \qquad r > 1$$

證明系統為不穩定

假設 $|x[n]| \le M_x < \infty$ for all n

$$|y[n]| = |r^n x[n]|$$

$$= |r^n| \cdot |x[n]| \xrightarrow{n \to \infty} = \infty$$

$\therefore$ 系統為不穩定

Memory: 記憶性

ee:p57

- 若一個系統之輸出與過去或未來之訊號有關, 則稱系統有記憶性

$$i(t) = \frac{1}{R} v(t) \longrightarrow memoryless$$

$$i(t) = \frac{1}{L} \int_{-\infty}^{t} v(\tau) d\tau \longrightarrow memory$$

$$y[n] = \frac{1}{3} \left(x[n] + x[n-1] + x[n-2] \right) \longrightarrow memory$$

Problem

ee:p58

$problem 1.27$

$$y[n]=\frac{1}{3}\left(x[n]+x[n-2]+x[n-4]\right)$$

使用過去訊號
4個時間單位

有使用過去時間訊號?

$problem 1.28$ $\quad i(t)=a_0+a_1v(t)+a_2v^2(t)+a_3v^3(t)+\ldots$

有memory?

只與t時之訊號有關, **無記憶**

$problem 1.29$ $\quad v(t)=\frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$

有memory?

使用過去至t之訊號, **有memory**

Causality: 因果性

- 若一個系統之輸出只與過去與現在訊號有關, 則稱系統為因果系統

過去時間

$$y[n]=\frac{1}{3}\left(x[n]+x[n-1]+x[n-2]\right)$$

→為因果系統

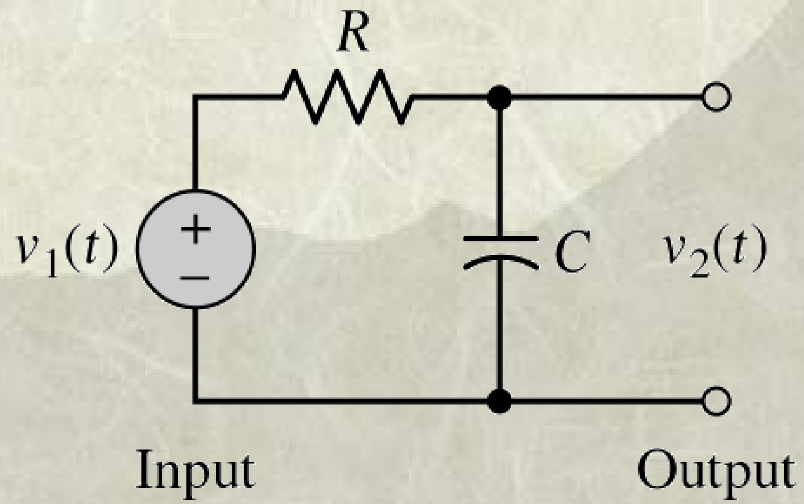
未來時間

$$y[n]=\frac{1}{3}\left(x[n+1]+x[n]+x[n-1]\right)$$

→為非因果系統

Problem 1.30 下圖系統為因果或非因果?

ee:p59



$$v_2(t) + RC\frac{d}{dt}v_2(t) = v_1(t)$$

與未來輸入無關 $\therefore$ *causal*

ee:p59

Invertibility: 反系統

- Invertible: 若系統之輸入可以由輸出還原稱此系統具反系統

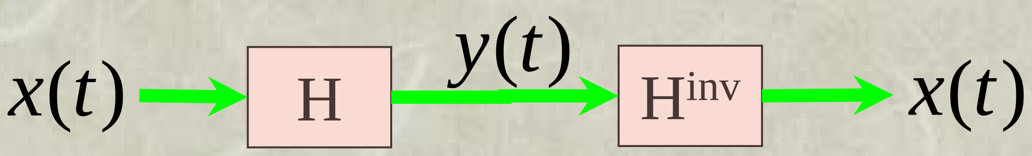
$$y(t) = H\{x(t)\}$$

$$\rightarrow H^{inv}\{y(t)\} = x(t)$$

$$\rightarrow H^{inv}\{H\{x(t)\}\} = x(t)$$

$$\rightarrow H^{inv}H\{x(t)\} = x(t)$$

必需 $H^{inv}H = 1$



E 1.15 求$y(t)=x(t-t_0)=S^{t_0}\{x(t)\}$之反系統

ee:p60

$$x(t)=x(t-t_0+t_0)=x((t-t_0)-(-t_0))$$

$$=S^{-t_0}\{x(t-t_0)\}=S^{-t_0}\{S^{t_0}\{x(t)\}\}$$

$$=S^{-t_0}\cdot S^{t_0}\{x(t)\}=I\{x(t)\}$$

$$\rightarrow S^{-t_0}\cdot S^{t_0}=I$$

$\therefore S^{-t_0}$為S^{t_0}之反系統

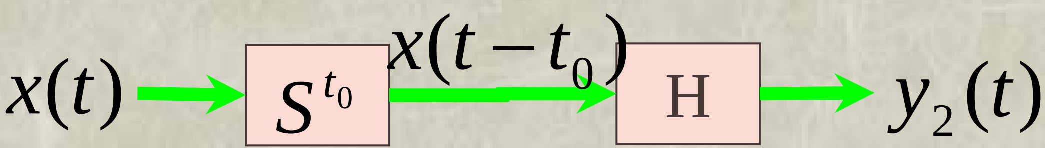
ee:p60

Time invariance: 非時變(1)

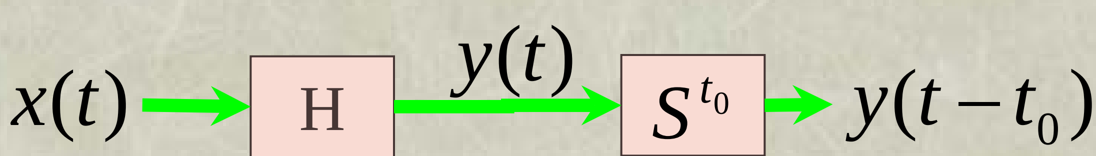
- 若一個系統符合下列條件稱非時變

$$y(t) = H\{x(t)\}$$

$$H\{x(t-t_0)\} = y(t-t_0)$$



$$\equiv \qquad y_2(t) = y(t-t_0)$$



ee:p61

Time invariance: 非時變(2)

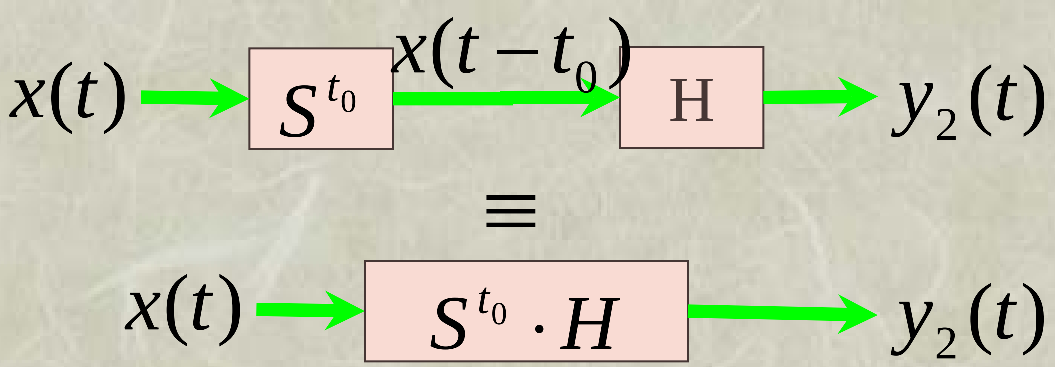
- 若系統H為非時變則此系統與shift運算有交換律

$$y(t)=H\{x(t)\}$$

$$H\{x(t-t_0)\}=H\{S^{t_0}\{x(t)\}\}=H\cdot S^{t_0}\{x(t)\}$$

$$y(t-t_0)=S^{t_0}\{y(t)\}=S^{t_0}\{H\{x(t)\}\}=S^{t_0}\cdot H\{x(t)\}$$

$$\rightarrow H\cdot S^{t_0}\equiv S^{t_0}\cdot H$$



ee:p61~62

Ex 1.17 $y(t)=\frac{1}{L}\int_{-\infty}^{t}x(\tau)d\tau$ 是否時變？

$$y(t)=H\{x(t)\}=\frac{1}{L}\int_{-\infty}^{t}x(\tau)d\tau$$

$$H\{x(t-t_0)\}=\frac{1}{L}\int_{-\infty}^{t}x(\tau-t_0)d\tau \qquad (t'=\tau-t_0,\ dt'=d\tau)$$

$$=\frac{1}{L}\int_{-\infty}^{t-t_0}x(t')dt' \qquad (\tau\text{原上限}t,\ t'\text{之上限}\tau-t_0=t-t_0)$$

$$=y(t-t_0)$$

ee:p62

x 1.18 $y(t)=\frac{x(t)}{R(t)}$ 是否時變？

$$y(t)=H\{x(t)\}=\frac{x(t)}{R(t)}$$

$$H\{x(t-t_0)\}=\frac{x(t-t_0)}{R(t)}=y_1(t)$$

$$y(t-t_0)=\frac{x(t-t_0)}{R(t-t_0)}=y_2(t)$$

$$y_1(t)\neq y_2(t)$$

$\therefore$ time varing

ee:p63

Linearity: 線性

- 若系統H符合重疊性(superposition)與齊次性(Homogeneity), 則稱H為線性系統
- Superposition:

$$y(t) = H\{x(t)\}, \qquad y_i(t) = H\{x_i(t)\}, \qquad i = 1 \ldots N$$

$$\text{if} \quad \sum_{i=1}^{N} y_i(t) = H\left\{\sum_{i=1}^{N} x_i(t)\right\} \quad \textit{then}\ \text{H has suposition}$$

- Homogeneity:

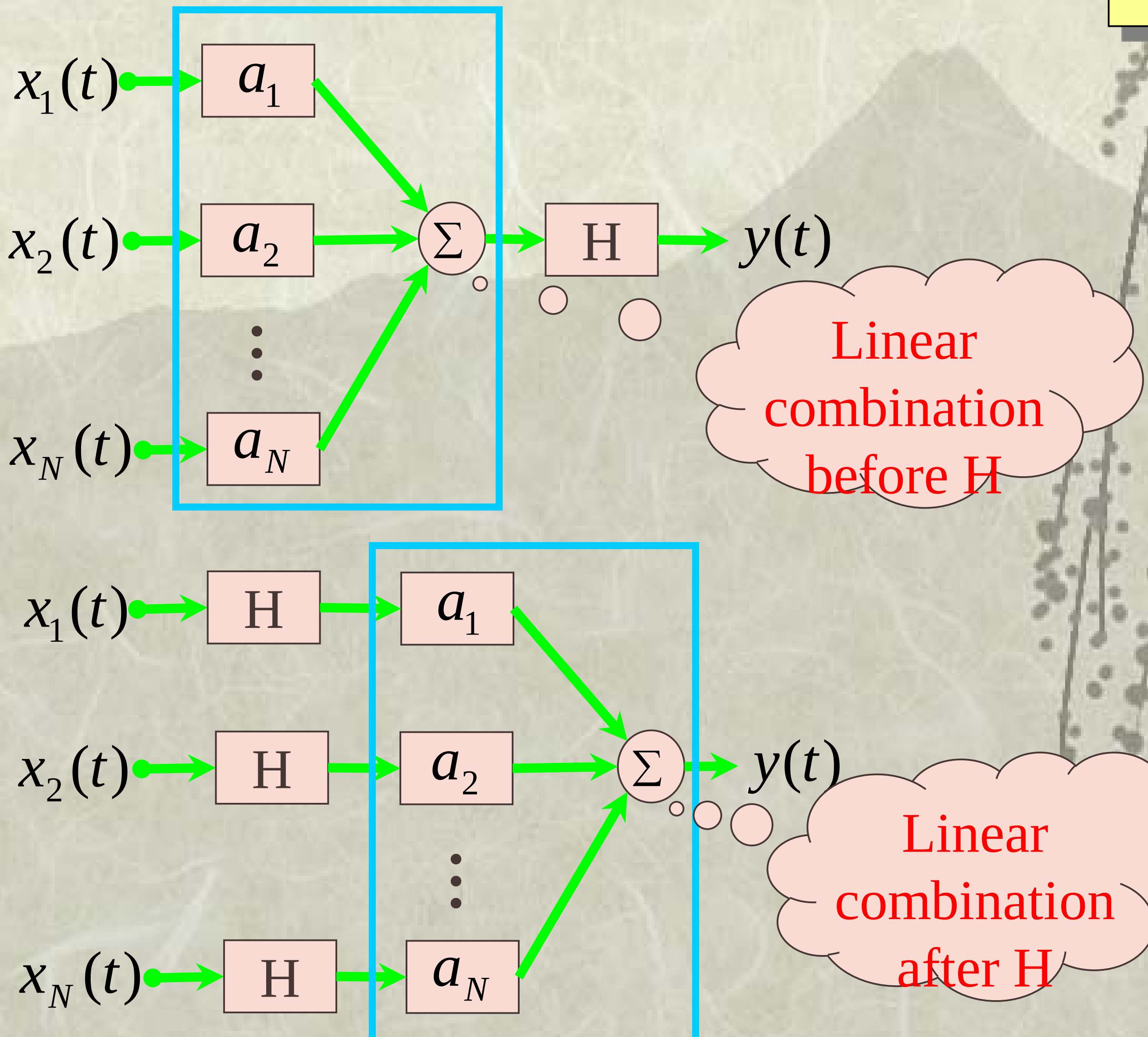
$$y(t) = H\{x(t)\}, \qquad \text{if} \quad ay(t) = H\{ax(t)\} \quad \textit{then}\ \text{H is homogent}$$

- Linearity:

$$\text{if} \quad \sum_{i=1}^{N} a_i y_i(t) = H\left\{\sum_{i=1}^{N} a_i x_i(t)\right\} \quad \textit{then}\ \text{H is linearity}$$

The Linearity Property of a System.

ee:p64



Ex1.19 證明 y[n]=nx[n] 為線性.

ee:p64

$$operator\ H:\ H\{x[n]\} = y[n] = nx[n]$$

$$y[n] = H\{x[n]\}, \qquad y_i[n] = H\{x_i[n]\}, \qquad i = 1...N$$

$$\sum_{i=1}^{N} y_i[n] = \sum_{i}^{N} H\{x[n]\} = \sum_{i}^{N} nx_i[n]$$

$$= n\left(\sum_{i}^{N} x_i[n]\right) = H\left\{\sum_{i}^{N} nx_i[n]\right\} \qquad \therefore \text{H has suposition}$$

$$ay[n] = aH\{x[n]\} = anx[n]$$

$$= nax[n] = H\{ax[n]\} \quad \therefore \text{H is homogent}$$

- H is linearity

Ex1.20 證明 y(t)=x(t)x(t-1) 為非線性.

ee:p65

$$operator\ H:\ H\{x(n)\} = y(t) = x(t)x(t-1)$$

$$y(t) = H\{x(t)\}, \qquad y_i(t) = H\{x_i(t)\} = x_i(t)x_i(t-1), \qquad i = 1 \ldots N$$

$$\sum_{i=1}^{N} y_i(t) = \sum_{i=1}^{N} H\{x_i(t)\} = \sum_{i=1}^{N} x_i(t)x_i(t-1)$$

$$\neq H\left\{\sum_{i=1}^{N} x_i(t)\right\} = \left(\sum_{i=1}^{N} x_i(t)\right)\left(\sum_{i=1}^{N} x_i(t-1)\right)$$

$$ay(t) = aH\{x(t)\} = ax(t)x(t-1)$$

$$H\{ax(t)\} = (ax(t))(ax(t-1)) = a^2 x(t)x(t-1)$$

$$\rightarrow ay(t) \neq H\{ax(t)\}$$

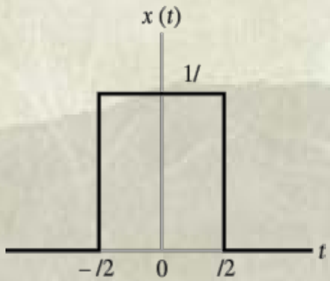
- H 不符合superposition & homogeneity.
- H為非線性

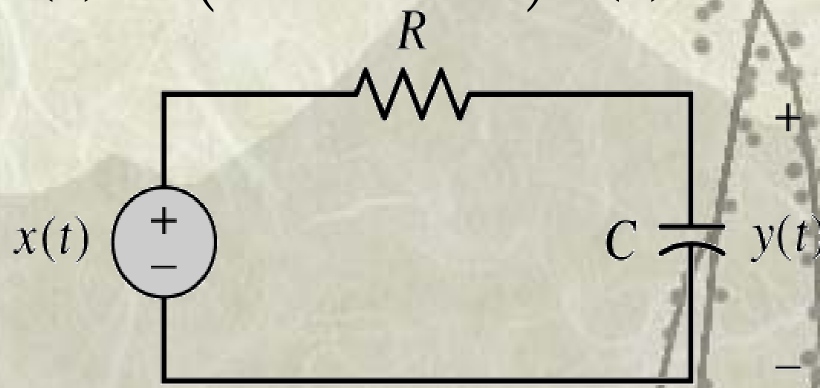
Ex 1.21 Impulse Response of RC (1)

ee:p65~67

$x(t)=u(t)$,右圖之解 $\quad y(t)=\left(1-e^{-(t/RC)}\right)u(t)$

求$x(t)=\delta(t)$之解





$$operator\ \mathrm{H}: H\{x(t)\}=\left(1-e^{-(t/RC)}\right)x(t)$$

假設H為線性非時變

令 $\quad x_1(t)=\frac{1}{\Delta}u\left(t+\frac{\Delta}{2}\right),\quad x_2(t)=\frac{1}{\Delta}u\left(t-\frac{\Delta}{2}\right)$

則 $\quad x(t)=\delta(t)=\lim_{\Delta\to 0}\left(x_1(t)-x_2(t)\right)$

Ex 1.21 Impulse Response of RC (2)

ee:p65~67

$$y_1(t) = H\left\{\frac{1}{\Delta}x\left(t+\frac{\Delta}{2}\right)\right\} = \frac{1}{\Delta}\left(1-e^{-\left(\left(t+\frac{\Delta}{2}\right)/RC\right)}\right)u\left(t+\frac{\Delta}{2}\right)$$

$$y_2(t) = H\left\{\frac{1}{\Delta}x\left(t-\frac{\Delta}{2}\right)\right\} = \frac{1}{\Delta}\left(1-e^{-\left(\left(t-\frac{\Delta}{2}\right)/RC\right)}\right)u\left(t-\frac{\Delta}{2}\right)$$

$$y_\Delta(t) = H\{x_1(t) - x_2(t)\} = y_1(t) - y_2(t)$$

$$= \frac{1}{\Delta}\left(1-e^{-\left(\left(t+\frac{\Delta}{2}\right)/RC\right)}\right)u\left(t+\frac{\Delta}{2}\right) - \frac{1}{\Delta}\left(1-e^{-\left(\left(t-\frac{\Delta}{2}\right)/RC\right)}\right)u\left(t-\frac{\Delta}{2}\right)$$

$$y(t) = \lim_{\Delta\to 0} y_\Delta(t) = \delta(t) - \frac{d}{dt}\left(\left(e^{-(t/RC)}\right)u(t)\right)$$

$$= \delta(t) - \frac{d}{dt}\left(e^{-(t/RC)}\right)u(t) - e^{-(t/RC)}\delta(t) = \frac{1}{RC}e^{-(t/RC)}u(t)$$