An Overview of Structural Equation Modeling

Karl Jöreskog and LISREL: A personal story

- 1935, 4/25, born in Amal, Sweden
- Uppsala: 1955-1963
  "Ambition was to study mathematics and physics and become a high school teacher"
- Princeton: 1964-1971
  "Judge the magnitude of impact by the size of the waves. His influence, by any standard, is enormous… (Cudeck, Du Toit, Sorbom, 2001)

Growth of Structural Equation

Figure 2. Total number of articles and journals by year

Figure 3. Percentage of articles using different analytical methods

Structural equation model U

specifies the causal relationships among the latent variables, describes the causal effects, and assigns the explained and unexplained variance.

Softwares

- LISREL (Jöreskog & Sörbom, 1996)
- EQS (Bentler, 1995)
- CALIS (SAS)
- RAMONA (Browne & Mels, 1998, SYSTAT)
- AMOS (Aruckle, 1999, SPSS)
- SEPATH (Steiger, 1999, CSS STATISTICA)
- Mx (Neale, 1997)
- MECOSA (Arminger & Schepers)
An Overview of Structural Equation Modeling

- General Pre-requisites
  - Linear Regression Analysis
  - Probability, estimation and hypothesis testing
  - Basics of factor analysis
  - Basics of measurements (reliability and validity)

- Some Statistics Concepts
  - The good understanding of the following concepts is required:
  - Measurement levels (nominal, ordinal, interval and ratio)
  - Correlation and covariance
  - Indicators (reliability and validity)
  - Goodness fit indicator (R² …)

- OLS Regression
  - X1
  - X2
  - X3
  - Y
An Overview of Structural Equation Modeling

Moderator & Mediator

SES
Performance
Salary
IQ
Attitude

The full latent variable model
- Measurement model: depicts the links between the latent variables and their observed measures
- Structural model: depicts the links among the latent variables themselves.

Diagram Symbols
- Latent variables
- Observed variables
- Direct effects (Path)
- Correlations/ Covariances

Components
- the measurement model \( \text{MM} \)
- \( y = \Lambda_y \eta + \epsilon \) \( \text{SD}_1 \) \( \text{SD}_3 \)
- \( x = \Lambda_x \xi + \delta \) \( \text{SD}_2 \)
- the structural equation model \( \text{SEM} \) \( \text{ex55A_PD} \)

\[ \eta = B\eta + \Gamma \xi + \zeta \]

From Equations to Matrices

\[ \begin{align*}
\theta & \quad \Lambda_X \\
\delta_1 & \quad \xi_1 \\
\delta_2 & \quad \xi_2 \\
\delta_3 & \quad \xi_3 \\
\delta_4 & \quad \xi_4 \\
\delta_5 & \quad \xi_5 \\
\delta_6 & \quad \xi_6 \\
\end{align*} \]

\[ \begin{align*}
\eta_1 & \quad \zeta_1 \\
\eta_2 & \quad \zeta_2 \\
\end{align*} \]

\[ \begin{align*}
\Lambda_Y & \quad \theta_\epsilon \\
\Lambda_X & \quad \Phi \\
\Gamma & \quad B \\
\Psi & \quad \Lambda_Y \\
\end{align*} \]

\[ \begin{align*}
\delta_1 & \quad x_1 \\
\delta_2 & \quad x_2 \\
\delta_3 & \quad x_3 \\
\delta_4 & \quad x_4 \\
\delta_5 & \quad x_5 \\
\delta_6 & \quad x_6 \\
\end{align*} \]

\[ \begin{align*}
\epsilon_1 & \quad \xi_1 \\
\epsilon_2 & \quad \xi_2 \\
\epsilon_3 & \quad \xi_3 \\
\epsilon_4 & \quad \xi_4 \\
\end{align*} \]

\[ \begin{align*}
\delta_1 & \quad \zeta_1 \\
\delta_2 & \quad \zeta_2 \\
\delta_3 & \quad \zeta_3 \\
\delta_4 & \quad \zeta_4 \\
\end{align*} \]

\[ \begin{align*}
\epsilon & \quad \text{is uncorrelated with } \eta \\
\delta & \quad \text{is uncorrelated with } \xi \\
\zeta & \quad \text{is uncorrelated with } \xi \\
\zeta & \quad \text{is uncorrelated with } \epsilon \text{ and } \delta .
\end{align*} \]
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Step 2: Diagnostics 1

Modeling

Step 4: Variable Transformation

Step 3: Diagnostics 2 – linearity and outliers

Structural Model Equations

\[ \eta_1 = \gamma_{12} \xi_1 + \gamma_{11} \xi_2 + \zeta_1 \]

\[ \eta_2 = \beta_{21} \eta_1 + \zeta_1 \]

Measurement Model Equations

\[ x_1 = \lambda_{11} \xi_1 + \delta_1 \]

\[ x_2 = \lambda_{21} \xi_1 + \delta_2 \]

\[ x_3 = \lambda_{31} \xi_1 + \delta_3 \]

\[ x_4 = \lambda_{41} \xi_1 + \delta_4 \]

\[ y_1 = \lambda_{11} \eta_1 + \epsilon_1 \]

\[ y_2 = \lambda_{21} \eta_1 + \epsilon_2 \]

\[ y_3 = \lambda_{31} \eta_1 + \epsilon_3 \]

\[ y_4 = \lambda_{42} \eta_2 + \epsilon_4 \]

8 Matrices_1

8 Matrices_2

\[ \phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \]

\[ \psi = \begin{bmatrix} \psi_{11} \\ \psi_{21} & \psi_{22} \\ \psi_{31} & \psi_{32} & \psi_{33} \end{bmatrix} \]

\[ \Theta = \begin{bmatrix} \delta_{11} & 0 & 0 & 0 \\ 0 & \delta_{22} & 0 & 0 \\ 0 & 0 & \delta_{33} & 0 \\ 0 & 0 & 0 & \delta_{44} \end{bmatrix} \]

\[ \Theta_0 = \begin{bmatrix} \epsilon_{11} \\ 0 & \epsilon_{22} \\ 0 & 0 & \epsilon_{33} \\ 0 & 0 & 0 & \epsilon_{44} \end{bmatrix} \]

Step Approach to Build a Regression Model - 1

- Step 1: Data Cleaning & First OLS Results – missing data, added variable plots, dummy variables, collinearity, first OLS estimation
- Step 2: Diagnostics 1 – normality, homoscedasticity
- Step 3: Diagnostics 2 – linearity and outliers (residual plot and studentized residuals)
- Step 4: Variable Transformation

- Step 5: Model Assessment & Validation – variable selection & model validation (step-wise and cross-validation)
- Step 6: Diagnostics Again – if problems go back to step 4
- Step 7: Final OLS Estimates – if necessary, use non-OLS methods
**Applications in Social Science**

**Other Applications**
- Testing for construct validity: the multitrait-Multimethod Model
- Testing the validity of a causal structure
- Multiple Group Analyses
- Testing for causal predominance using a two-wave panel model

**Path Analysis**

**1st Order CFA Model**


**2nd Order CFA Model**


**Multiple Sample Analysis**


Testing for causal predominance using a 2-wave panel model

GSC: general self-concept
ASC: academic self-concept
AA: academic achievement

Byrne BM (1998). Structural equation modeling with LISREL, PRELIS, and SIMPLIS. LEA (p. 353)

References

- **Major Textbooks**:
- **Supplementary Books**:

Thank You
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